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II. Solution by E. B. ESCOTT, Ann Arbor, Mich.

Call the given equation $Mdx+Ndy=0\ldots(2)$.
If F is an integrating factor, we must have

$$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=\frac{N\dfrac{\partial F}{\partial x}-M\dfrac{\partial F}{\partial y}}{F}\ldots(3).$$

From symmetry of M and N, we may suppose F to be symmetrical in x and y. Suppose F is a function of $x+y$. Then

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}=\frac{\partial F}{\partial z},$$

and equation (3) becomes an ordinary differential equation instead of a partial differential equation. (3) becomes when the values of M and N are substituted,

$$(4a_1-3a_0-3a_2)(x^2-y^2)+(a_4-2a_5)(x-y)=$$
$$-[(a_1-a_2)(x+y)^2+(a_3-a_5)(x+y)+a_6-a_7](x-y)\frac{\frac{dF}{dz}}{F},$$

or, dividing by $x-y$ and putting $x+y=z$,

$$(4a_1-3a_0-3a_2)z+a_4-2a_5=-[(a_1-a_2)z^2+(a_3-a_5)z+a_6-a_7]\frac{dF}{dz}\cdot\frac{1}{F}.$$

The variables are separable, and we have

$$\frac{dF}{F}=-\frac{(4a_1-3a_0-a_2)z+a_4-2a_5}{(a_1-a_2)z^2+(a_3-a_5)z+a_6-a_7}dz=\left(\frac{a}{z-\alpha}+\frac{b}{z-\beta}\right)dz.$$

Then $F=(z-\alpha)^a(z-\beta)^b$, etc.

291. Proposed by V. M. SPUNAR, M. and E. E., Pittsburg, Pa.

Integrate $\frac{dy}{dx}=ay^2+bx^m$.

Solution by **J. SCHEFFER, A. M.,** Hagerstown, Md.

This is the famous Riccati equation. It can be solved only under certain circumstances.

If $m=0$, we have $\dfrac{dy}{ay^2+b}+dx=0$, easily integrated. If $m=-2$, $\dfrac{dy}{dx}=ay^2+\dfrac{b}{x^2}$ becomes homogeneous, if y is replaced by z^{-1}. If $m=-4$, we put $y=\dfrac{1}{ax}+\dfrac{z}{x^2}$, then we have $\dfrac{dz}{az^2+b}+\dfrac{dx}{x^2}=0$, which can easily be integrated. When m has the form $-\dfrac{4n}{2n-1}$ or $-\dfrac{4n}{2n+1}$ the equation yields integrable forms, putting in the former case $x=1/u$, in the latter $y=1/z$.

This problem is fully treated in Johnson's *Differential Equations*, Chapter IX, and in Forsyth's *Differential Equations*, second edition, pages 170-176. A number of our contributors referred to these and other sources. Ed.F.

MECHANICS.

240. Proposed by **S. A. COREY,** Hiteman, Iowa.

A perfectly flexible wire rope weighing one pound per foot is suspended from the tops of two vertical supports 300 feet apart, one support being 30 feet higher than the other. One end of the rope is fastened to the top of the higher support, while 600 feet of the rope hangs vertically from the top of the lower support. Assuming that the rope is free to slide over the top of the lower support without friction, find the lowest point of that portion of the rope which is suspended between the supports. Also find the amount of work which must be performed in raising the lowest point to make it coincide with the top of the lower support by exerting a pull on the free end of the rope.

Discussion by **F. H. SAFFORD, Ph. D.,** University of Pennsylvania.

This problem involves several points of theoretical interest which may be better treated as a general problem. The analysis will be carried to such a stage that the transcendental equations involved may be used to obtain numerical results by any convenient method of computation.

It is evident that between the supports the rope will hang in a catenary which, if referred to appropriate axes to be found, has the equation

$$y=c\cosh\frac{x}{c}. \tag{1}$$

Let the tops of the two supports be $(-a, b)$ and (a_1, b_1), and let the free end of the rope hang from the first point.

"The tention at any point of the catenary is equal to the weight of a portion of the string whose length is equal to the ordinate of the point" (Minchin's *Statics, or* Bowser's *Analytic Mechanics*). With this use of the term "ordinate" the X-axis must be the directrix of the catenary, and since in this problem the rope is of unit length, weights and lengths are numeric-